# Triple charged-particle decays of resonances illustrated by $^{12}$C-states

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**Abstract.** The hyperspherical adiabatic expansion is combined with complex scaling and used to calculate the energy distributions of the particles arising from three-body decaying low-lying $^{12}$C resonances. The large distance continuum properties of the wavefunctions are crucial and must be accurately calculated. The substantial changes from small to large distances determine the decay mechanisms. We illustrate by computing the energy distributions from decays of the $1^-$, $2^-$ and $4^-$-resonances in $^{12}$C. These states are dominated by sequential ($1^-$), through the $^8$Be ground state, and direct ($2^-$, $4^-$) decays.

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## 1. Introduction.

The importance of the triple-$\alpha$ process is well known in nuclear astrophysics. The process leads from three free $\alpha$-particles via a low-lying resonance into the ground state of $^{12}$C. The inverse of the crucial part of this process is the decay of a resonance of $^{12}$C into the continuum of three $\alpha$-particles, perhaps via intermediate states of $^8$Be [1]. The principle of detailed balance relates direct and inverse processes. Similar processes involving three charged particles occur at specific waiting points for the *rp*-process [2].

Moreover, the breakup of a physical system into a three-body continuum with Coulomb interaction is not yet a well understood problem of few body physics, although it has been studied over many years. In the three-body final state the asymptotics are determined by the dynamics of the breakup process itself. The study of these processes would help us to find out about the inverse process (triple-$\alpha$ and 2p capture) and to study to what extent the three particles of the final state may be present as a cluster structure in the many-body initial state. The main difficulty lies in constructing correct asymptotic wave functions when both two- and three-body structures are present.

Several published experimental results strongly suggest that the decay mechanism varies from sequential decay, via an intermediate quasi-stable state, to direct decay

into the three-body continuum [1]. The corresponding energy distributions exhibit completely different shapes. Reliable model computations reproducing the measured energy distributions are not available. This may be understandable due to several indispensable difficult requirements, but it is nevertheless very unfortunate because quantitatively accurate models are needed to extract and understand the underlying physics contained in the increasing amount of experimental high-quality data sets.

In the present paper we exploit a reliable recently developed technique to compute the large distance properties of a many body resonance that decays into three particles. We shall use the hyperspherical adiabatic expansion method of the Faddeev equations combined with complex scaling of the coordinates [3]. The method first provides an accurate computation of the resonance properties, which determines the energy distribution between the three particles after the decay.

## 2. Theoretical framework.

We describe $^{12}$C as a three-$\alpha$ cluster system, since triple-$\alpha$ decay is the only open non-electromagnetic decay channel for this nucleus. We use the hyperspherical adiabatic expansion method combined with complex scaling to compute the resonance wavefunctions [3]. The so-called hyperradius $\rho$ is the most important of the coordinates, and is defined as

$$m_N \rho^2 = \frac{m_\alpha}{3} \sum_{i<j}^{3} (\boldsymbol{r}_i - \boldsymbol{r}_j)^2 = m_\alpha \sum_{i=1}^{3} (\boldsymbol{r}_i - \boldsymbol{R})^2 \ , \tag{1}$$

for three identical particles of mass $m_\alpha = 4m_N$, where we choose $m_N$ to be equal to the nucleon mass, $\boldsymbol{r}_i$ is the coordinate of the i-th particle and $\boldsymbol{R}$ is the centre-of-mass coordinate.

We first choose the two-body interaction to reproduce the low-energy two-body scattering properties [4]. To adjust the energy position of the many-body resonance we add then a three-body potential, whose range corresponds to three touching $\alpha$-particles. These potentials are chosen independently for each $J^\pi$.

The total wave function is expanded on the angular eigenfunctions $\Phi_{nJM}(\rho, \Omega)$ obtained as solutions to the Faddeev equations for fixed $\rho$,

$$\Psi^{JM} = \frac{1}{\rho^{5/2}} \sum_n f_n(\rho) \Phi_{nJM}(\rho, \Omega) \ , \tag{2}$$

where the radial coefficients $f_n(\rho)$ are obtained from the coupled set of radial equations. The effective adiabatic potentials are the eigenvalues of the angular part.

The three-body potential is used to adjust the small-distance part of the effective potential in order to reproduce the correct resonance energies; at intermediate distances the potential has a barrier that determines the resonance width; and at large distances the resonance wave functions contain information about distributions of relative energies.

The large-distance asymptotic behaviour of the decaying resonance wave function determines the observable energy distribution. This energy distribution can be computed in coordinate space, except for a phase-space factor, as the integral of the absolute square of the total wave function for a large value of the hyperradius [5]. We shall explore the conjecture that the final state can be described entirely within the present cluster model.

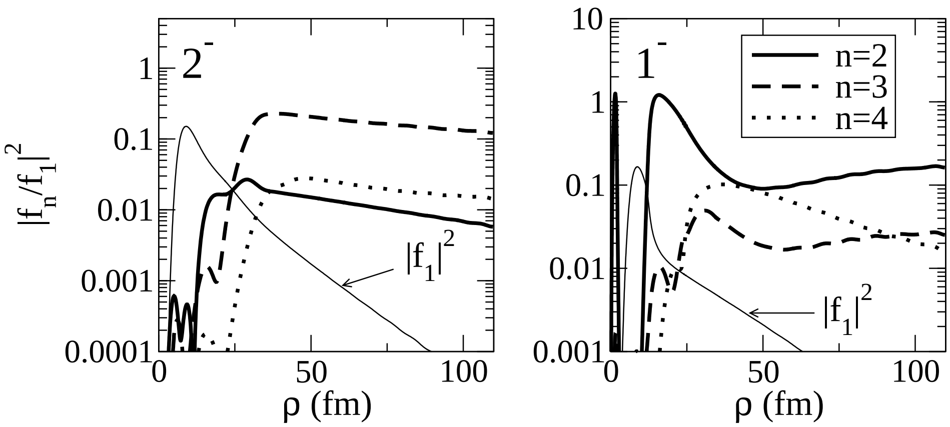


**Figure 1.** Ratios between the four lowest adiabatic radial wave functions and the small-distance dominating wavefunctions as a function of $\rho$ for the non-natural-parity state (i.e. angular momentum conservation forbid the contribution of the ${}^8$Be($0^+$) resonance) $2^-$ (to the left) and the natural-parity state $1^-$ (to the right) of ${}^{12}$C.

## 3. Results.

The method allows investigations of a number of the low-lying ${}^{12}$C resonances, i.e., two $0^+$, three $2^+$, two $4^+$, and one of each of $1^\pm$, $2^-$, $3^\pm$, $4^-$ and $6^+$ [6], all below 13.5 MeV.

The resonance structures are changing substantially from small to large distances. The crucial observables to study the dynamics of the breakup process are the momentum distributions of the particles after the decay. We use the $2^-$, $4^-$ and $1^-$ resonances of ${}^{12}$C as examples in this report. In our computation we get only one $2^-$ and one $4^-$ state. We suggest the latter should correspond to the state at 6.08 MeV above the triple-$\alpha$ threshold for which a preliminary spin-parity of $2^-$ is assigned in [7]. The computed energy distributions could also be used as a tool to decide this assignment by comparing with measurement. The energies and widths are compatible with the experimental values. For $1^-$ we get $(E_R,\Gamma_R)$=(3.61,0.475) MeV, compared to the experiment (3.57,0.315) MeV; for $2^-$ we get (4.53,0.452) MeV, being the experimental values (4.55,0.260) MeV; and for the uncertain $2^-/4^-$ the theoretical values are (5.98,1.035) MeV and the experimental ones (6.08,0.375) MeV.

In ref. [8] we have shown the results corresponding to the $0^+$ and $1^+$ states of ${}^{12}$C. In both cases accurate measurements of $\alpha$-particle energy distributions are available [9]. The $0^+$ resonances are often approximated by $\alpha$-cluster states preferentially decaying sequentially. In contrast the $1^+$ resonance has no significant cluster structure and is referred to as a shell-model state. Its decay has been suggested to be direct. Still for both cases the decays into three $\alpha$-particles are determined at intermediate and large distances. The agreement with the accurately measured distributions is remarkably good. We conclude that the $1^+$ resonance is best described by direct decay into the three-body continuum, whereas the $0^+$ resonances decay sequentially via ${}^8$Be ground state.

We show in fig.1 the ratios of the four lowest adiabatic radial wavefunctions as

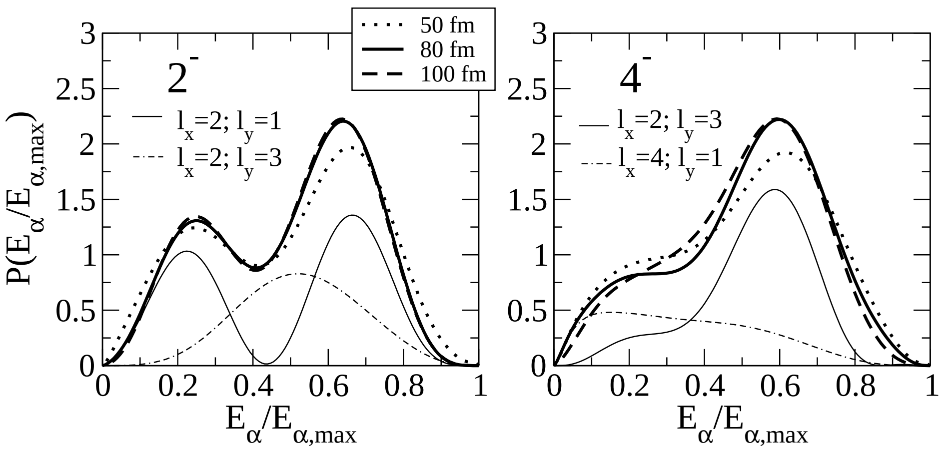


**Figure 2.** To the left, the $\alpha$-particle energy distribution for the $2^-$ resonance of $^{12}$C at 11.8 MeV (or 4.53 MeV above the threshold). To the right, the $\alpha$-particle energy distribution for the $4^-$ resonance of $^{12}$C at 13.3 MeV (or 5.98 MeV above the threshold).The hyperradii are $\rho = 50, 80, 100$ fm. The contribution of the most important partial waves is also shown. The energy is measured in units of the maximum possible.

a function of the hyperradius for the $2^-$ and $1^-$ resonances located at 4.5 MeV and 3.6 MeV above the triple-$\alpha$ threshold, respectively. One can note from the figure that only the component relative to the first adiabatic potential contributes significantly. This reflects the fact that only few eigenvalues are needed to get an accurate solution, due to the fast convergence of the adiabatic expansion. The relative sizes of the radial wavefunctions change dramatically around 25 fm and 15 fm for the $2^-$ and $1^-$ states, respectively. The corresponding change of structure around 30 fm is related to an attempt of populating the 2-body state $^8$Be($2^+$). The ratios of the radial wavefunctions are also shown to be quite stable under variations of the hyperradius at large distances, that is where the energy distributions are determined.

In fig.2 we plot the $\alpha$-particle energy distribution of the $2^-$ and $4^-$ resonances of $^{12}$C for different relatively large $\rho$-values. The asymptotic behaviour is already reached for hyperradii around 70 fm. It can be seen in fig.2 that there is a small variation of the $4^-$ distribution from 80 to 100 fm. In the $2^-$ distribution the curves corresponding to 80 and 100 fm are in fact indistinguishable. As a test we have Fourier transformed the wavefunction by use of the analytic solution to the numerically computed adiabatic potentials, which at large distances is approximated by sums of $1/\rho$ and $1/\rho^2$ terms. We obtain very similar distributions with both procedures. Since both $2^-$ and $4^-$ are non-natural parity states, their decay can not occur sequentially via $^8$Be ground state. We are dealing then with a direct decay into the three-body continuum, or perhaps alternatively with sequential decay via $^8$Be($2^+$). If we estimate the sequential decay via the $^8$Be($2^+$) it would produce a peak around $0.3E_{\alpha,max}$ in the $2^-$ and around $0.4E_{\alpha,max}$ in the $4^-$ resonance. These peaks do not correspond to the ones shown in fig.2, which means that the $2^-$ and $4^-$ energy distributions in the figure are not consistent with a sequential decay. The structure in fig.2 can be understood by separating into individual partial wave contributions. The two peaks for the $2^-$ state are then related to the two dominating terms of $l_x = 2$ and $l_y = 1, 3$, where $l_x$ and $l_y$

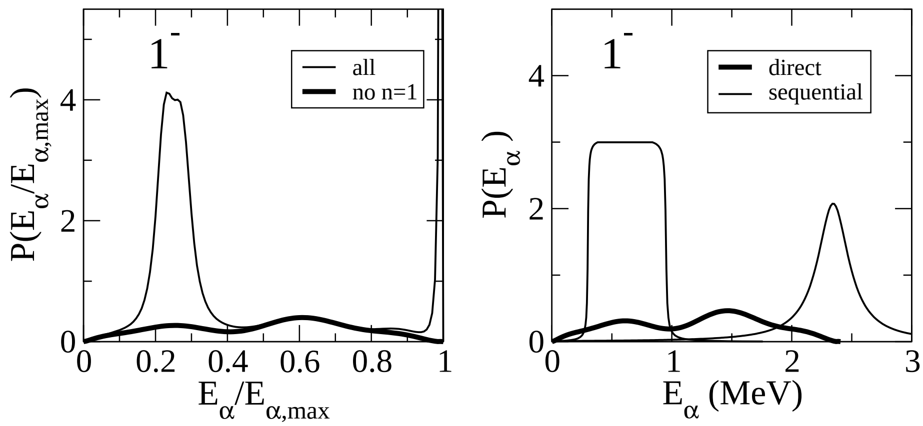


**Figure 3.** The $\alpha$-particle energy distribution for the $1^-$ resonance of $^{12}$C at 10.89 MeV (or 3.61 MeV above the threshold). To the left, the distribution is computed in coordinate space at $\rho = 80$ fm. The thick line correspond to the energy distribution without the contribution from the first adiabatic component. Its contribution is about 24% of the total. To the right, we keep the contribution without the first adiabatic component, that represents the direct decay. The sequential decay is illustrated by a Breit-Wigner shape.

are the angular momentum between two particles ($l_x$), and their centre of mass and the third particle ($l_y$). For the $4^-$-state both ($l_x$,$l_y$)=(2,3) and (4,1) contribute each with rather asymmetric distributions. Thus we describe these decays as direct into the three-body continuum.

The energy distribution for the $1^-$ resonance is shown in fig.3. The narrow high-energy peak and the distribution around one quarter of the maximum energy reflect the characteristic features of a sequential decay via a quasi-stable state. Since the two-body asymptotic behaviour has not been reached for a hyperradius smaller than 100 fm, the computed distributions are not accurate, but they provide us with information about the sequential decay. We use the fact that the sequential decay assymptotically must be contained in one of the adiabatic potentials, in the present case in the first one. We can substitute then the inaccurate component by the Fourier transform of the known asymptotic two-body behaviour. This sequential part of the energy distribution is approximated by the leading order Breit-Wigner shape for the first emitted $\alpha$-particle. It has a high-energy peak at the most probable position (2/3 of the resonance energy). The width is the sum of the three-body decaying resonance width and the width of the intermediate two-body resonance. By kinematic conditions we can compute the energy of the two $\alpha$-particles emerging from $^8$Be, that gives rise to the peak at lower energy. After removing the contribution from the first adiabatic potential, the remaining energy distribution is described as direct decay by the computed three-body continuum coordinate space wavefunction. From the results in fig.3 we find that the direct decay is about 24% of the total distribution. The distribution is rather uniform but mostly visible between the separate peaks of the sequential decay.

## 4. Summary and Conclusions.

We have applied a general new method to compute the particle-energy distributions of some of the three-body decaying $^{12}$C many-body resonances. We conjectured that the energy distributions of the decay fragments are insensitive to the initial many-body structure. The energy distributions are then determined by the energy and three-body resonance structure as obtained in a three-$\alpha$ cluster model. These momentum distributions are determined by the coordinate space wavefunctions at large distances.

We have shown the examples of $2^-$, $4^-$ and $1^-$ states of $^{12}$C. The preliminary spin-parity assignment of the $2^-$ state at 6.08 MeV above the triple-$\alpha$ threshold is suggested to be changed to $4^-$. Both $2^-$ and $4^-$ resonances are best described by direct decay into the three-body continuum, whereas the $1^-$ resonance have substantial cluster components and decays preferentially via $^8$Be ground state. The partial wave structure of the resonance at large distances is crucial for the energy distribution.

In conclusion, we predict energy distributions of particles emitted in three-body decays. Both sequential and direct, and both short and long range interactions are treated.

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